ELDERLY ASSET MANAGEMENT

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Introduction

We study asset management by elderly households in the last 20 years of life.

We focus on the effects of health status and bequest motive on asset management.

Elderly own more than half of all assets in U.S., many countries. Different factors may influence their decision-making about assets than for other age groups. Important for policy to study and understand these.

Structural model, simulation, estimation (to come).
Contribution

- 3 types of assets, cash, stock, house equity. Most previous life-cycle literature, especially focusing on elderly and incorporating health, incorporates only safe asset. We can study the effects of health status and changes in health on allocation of wealth between safe (cash) and risky (stock) asset types.

- Shape of bequest function. Bequest function does not have to have same shape as utility function. Can be more like risk-neutral (providing “extra” for children) or more risk-averse (leave nest egg).

- Incorporate impact of wealth on health, and health expenditure model.
Genesis

As I study creative development....


● J. Feinstein work on “The Relationship Between Socioeconomic Status and Health” (*The Milbank Quarterly*, 1993). Transition to health and wealth focus of interest.


● Ongoing work with Ching-Yang Lin on dynamic life-cycle modeling and simulation.
Previous Literature

There is a vast literature on life-cycle modeling stretching back to Modigliani (in fact to Walras). Most of this literature focuses on savings in terms of a riskless asset, and the entire life-cycle. Does not focus on elderly, thus not on health, and does not incorporate multiple asset types.

Important recent work: Hubbard, Skinner, Zeldes (JPE, 1995) – health and medical expenditure; focus on earlier part of life-cycle and lower end of income dist. Recent work on asset types and life cycle: Cagetti (JBusEcStat, 2003); Cocco, Gomes, and Maenhout (RevFinStud, 2005).

Work on the important of the bequest motive (for example Kotlikoff and Summers, Kotlikoff). We focus on shape of bequest function, not prior focus.

Model

We study a household/individual who lives for at most 7 periods. Each period is roughly 3 years. When the individual dies his/her remaining assets are included in a bequest given to heirs.

Each period that he is alive the individual is in either Good or Poor health.
Utility Function:

In each period the individual is alive his utility is given by:

\[ U = (C - C_0 - HX)^\alpha \]

This is standard CRRA form, with \( \alpha \) coefficient of relative risk aversion.

In this formula \( C_0 \) is “necessary expenditure” (original Stone-Geary form) important for the elderly.

HX is health expenditure, incurred only if household is in poor health, according to distribution given later.
Bequest

The shape of the bequest function is:

\[ B = BWT \cdot (W + \text{House})^\gamma \]

The Bequest function thus has a similar shape to the utility function. However, \( \gamma \) does not have to be the same as \( \alpha \). In particular, if \( \gamma \) is closer to 1 then the household treats bequest as more like risk-neutral. Interpretation: bequest is extra resources for children (heirs), not to be relied on. Alternatively, if \( \gamma \) is closer to 0 the household treats bequest as more risk-averse. Interpretation: wants to leave a nest-egg of modest amount to children, little value placed on leaving more.

BWT is a weighting function determining how much weight household puts on the bequest (compared with its own utility).
Income and Initial Wealth

The household has initial wealth $W_0$ at the start of period 1. Every period the household receives income $I$.

House

In some specifications the household also has a house with house equity. We assume (for now) it does not sell the house and does not draw down its house equity while alive (reverse annuity mortgage), but that when individual dies, house equity is part of bequest.
Asset Management

Each period the household chooses how to allocate its wealth (income + cash assets + stock assets) between consumption and reinvestment in cash and stocks. We let $s$ represent its cash savings, and $x$ represent its stock investment. It cannot draw down its house equity (or increase it).

Consumption $C$ is then given by:

$$C = I + \text{Cash} + \text{Stock} - s - x.$$

Importantly, we constrain the household’s non-housing wealth not to go negative. Realistic for elderly in most cases (at least not too negative; approximation). Makes solving the model a challenge. (Also, means-tested HX: $HX = \text{Min} (2, I-(C_0+HX))$ and non-negative.)
Maximization

The household chooses its consumption and asset allocation decision to maximize the expected net present value (NPV) of lifetime utility.

In period t, expected lifetime utility is the sum of current utility plus the weighted average over 3 possible states next period. Suppose current health status Good.

\[ U_G(C-C_0) + \text{Prob}(\text{Good}^{t+1}|G,W) \times \beta EV^{t+1}_G + \]
\[ \text{Prob}(\text{Bad}^{t+1}|G,W) \times \beta EV^{t+1}_B + \]
\[ \text{Prob}(\text{Death}^{t+1}|G,W) \times \beta BWT \times (W+\text{House})^\gamma. \]

Here \( \beta \) is the discount factor, and \( V \) is the value function, indexed by the time period and state. \( V^{t+1} \) inside integral over risky asset return.
Asset Dynamics

Recall each period lasts 3 years.

We assume the (real) return on cash over the 3 year period is 6%.

We assume the stock return follows a log-normal distribution: \( \mu = 0.025, \sigma = 0.19885 \). We truncate the integral at 3.8 (gross return nearly 4 times). Implied mean (real) return is 13.2% (3 years).

We assume house equity follows a simple 3-point discrete distribution: with probability 1/3, equity increases 35% over the 3 years; with probability 1/3 equity increases 10%; with probability 1/3 equity decreases 10%. (Historical data approximately.)
The household begins in Good or Poor health. When an individual is in good health in period $t$ he/she has a probability of falling into poor health next period, as well as a probability of death. When he/she is in poor health we assume he/she has a probability of remaining in poor health and a probability of death (but cannot return to good health). These probabilities are worked out from previous empirical estimates using AHEAD and HRS data by Chih-Chin Ho and J. Feinstein. They vary based on sex, age, wealth.

When a household moves from good to poor health we say that its first period in poor health it is in “worse” health. This also enters into health transition probability equations.
Health Expenditures

When an individual is in good health he/she has 0 health expenditures.

When she is in poor health, either in period 1 or first period in poor health, health expenditure has a 3-point discrete distribution (matches HES data): probability $1/3$ HX=0, probability $1/3$ HX=2, probability $1/3$ HX=20.

Once an individual is in poor health, next period, if alive, his/her HX is (i) same as last period with probability 0.5; (ii) redrawn from original HX distribution with probability 0.5.
Solving the Model: Feasible Space to Search

Search on boundary and inside triangle: constrained maximization: no borrowing, each investment non-negative.
Simulation Method To Solve the Model

- First, find maximum on each boundary, and max of these.

- Then search inside: find max along line segment in direction of gradient pointing uphill at $e_0$; then max along line segment from $e_1$, et cetera, till converge to interior max at $e^*$. (If gradients on boundaries point outside, max at $e_0$.)
Dynamic Programming Solution

We solve the problem using dynamic programming approach, working backwards, estimating V in each period over a grid of wealth values. Then in earlier period, we use interpolation to find $V^{t+1}(W)$ for particular value of W: Find $W_i$ and $W_{i+1}$ such that $W_i < W < W_{i+1}$, then interpolate over $V^{t+1}_i$ and $V^{t+1}_{i+1}$.

Grids (distance between points scales by utility curvature):

- Period 6: 2,000 points.
- Period 5: 1,517 points.
- Period 4: 1,121 points.
- Period 3: 797 points.
- Period 2: 531 points.
Simulation Parameters

Relative risk aversion parameter for utility: 0.15.

Bequest risk aversion parameter either 0.15 or 1.0 (risk-neutral).

Bequest weight BWT = Discount rate = 6%.

I = 25. Initial W = 500 or lower. House = 0 (baseline).

C₀ = 15. HX = 0, 2, or 20 (initial probabilities 1/3, 1/3, 1/3; given Poor health last period probability 0.5 same HX this period, probability 0.5 redraw. (If I-(c₀+HX) < 2, set = 2 (income-based assistance)).

Return on riskless asset = 6% (3 years).

Return on risky asset (equity): log-normal dist., μ=.025, σ² = .1988; mean return approximately 13.2% (3 years).

House: some runs, = 200, following period: probability 1/3 35% return, probability 1/3 10% return, probability 1/3 10% loss.
Simulation Results:
Bequest and Utility Functions Both Risk Averse
Relative Risk Parameter $= .15$
BWT = 6.8; Initial Wealth $W_0 = \$500K$

Total Savings Each Period (Average)
Percentage of Savings in Stock (vs. Safe Asset)
Total Savings By Wealth: Period 1
Total Savings By Wealth: Period 4
Simulation Results:
Risk-Neutral Bequest Function

The relative risk parameter remains at .15 for the utility function. Initial wealth remains at $500K.

We now must use very different BWT weight for bequest function. Example, if wealth is $500K, then $(500)^{.15} = 2.54$ (utility) and $2.54/500 \approx .005$. In other words, the bequest, not being “shrunk” by power .15, is much larger value.

Thus: we scale bequest down by adjusting BWT. Most interesting results when BWT below .0037. Above this value all saving on stock.

Focus here on BWT = .0025.
Total Savings Each Period (Average)
Percentage of Savings in Stock

Result: Those in Poor Health lower percentage of savings in stock. Varies by age.
Unusual: Stock Saving by Wealth Level
Period 4

Income protection for health costs: risky investment for some low W values. Poor health slightly more in stock for given W.